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Journal of Fluids and Structures 26 (2010) 148–159

JOURNAL OF FLUIDS AND STRUCTURES

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Resonant frequencies in an elevated spherical container partially filled with water: FEM and measurement

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Received 6 October 2008; accepted 6 October 2009
Available online 13 January 2010

Abstract

In this paper, a numerical-experimental study of the overall dynamical response of elevated spherical tanks subjected to horizontal base motion is presented. The main objective is to gain insight in the physical response of this particular structural typology widely used in the petrochemical industry as liquefied petroleum gas (LPG) containers. In order to identify the natural frequencies of the modes that mainly contribute to the response, experimental free vibration tests on an elevated spherical tank model for different liquid levels were carried out. Next, a numerical model that takes into account the coupling between fluid and structure was developed and validated against the experimental results. A very good agreement between experimental and numerical results was obtained. The results obtained show the influence of liquid levels on natural frequencies and indicate that the sloshing has a significant effect on the dynamical characteristics of the analyzed system. In order to obtain a good representation of the overall dynamical behaviour of the system by means of a simplified lumped mass model, a minimum of three masses is suggested. Finally, appropriate names of these three masses are proposed in the present paper.

Keywords: Fluid–structure interaction; Spherical storage tanks; Free vibration response

1. Introduction

The dynamical behaviour of elevated liquid storage containers is mainly studied because of the interest in their response to seismic loads (e.g., in petrochemical industry) or in the connection with the structural integrity and reliability analysis of diverse shell components (e.g., in nuclear reactors).

In the last five decades, many researchers have considered the topic of dynamical behaviour of liquid-filled tanks, mainly on cylindrical and rectangular storage tanks. A significant amount of experimental and theoretical effort has been invested on studies concerned with the understanding and predicting the seismic behaviour of ground-supported cylindrical tanks. One of the principal works was published in the early 1960s by Housner (1963), who considered ground-supported cylindrical rigid tanks subjected to horizontal translation, and suggested that the dynamical response can be idealized as the contribution of an impulsive (bulging) mass rigidly attached to the container wall and a sloshing

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0889-9746/$ - see front matter
doi:10.1016/j.jfluidstructs.2009.10.002

mass (convective) that is connected to the wall by means of springs. The impulsive component was attributed to the part of the liquid that vibrates jointly with the tank, while the sloshing component, which was characterized by long-period oscillations, corresponds to the liquid around of the free surface. To represent these effects, Housner (1963) considered a model with two uncoupled masses and developed equations to compute the impulsive and sloshing liquid masses, along with their location above the tank base and the stiffness of the convective mass spring. Although, for practical design, only one convective mass is commonly considered, additional lumped-masses may also be included (Bauer, 1964; Livaoğlu and Doğangün, 2006). For this type of tank, Haroun and Housner (1981) developed a three-mass model that takes the tank-wall flexibility into account. Other works concerning the natural modes and frequencies of clamped-free vertical cylindrical storage tanks by means of experimental and numerical approaches are reported by Chiba et al. (1985), Mazuch (1996), Chiba (1992, 1993, 1994), by analytical procedures Tang (1994), Han and Liu (1994), and using a FE formulation by Gonçalves and Ramos (1996), Cho et al. (2001) and Virella et al. (2006).

With regard to dynamical analysis of ground-supported rectangular tanks, several works have been reported using numerical methods, such as Doğangün et al. (1996) and Kianoush and Chen (2006), while Koh et al. (1998) and Faltinsen et al. (2005) included experimental tests. A recent work by Livaoğlu (2008), evaluates the dynamical behaviour of fluid–rectangular-tank–soil–foundation system with a simple seismic analysis procedure, based on Housner's two-mass approximations. Other investigations on soil–fluid–structure interaction effects for laterally excited tanks have been reported by Veletsos and Tang (1990) and Rammerstorfer et al. (1990).

The sloshing in this type of container has attracted some interest too (Chount and Yun, 1996; Faltinsen and Timokha, 2002; Faltinsen et al., 2003; Celebi and Akyildiz, 2002; Papaspyrou et al., 2004), as well as for applications in tuned liquid dampers to mitigate the dynamical response of structures subjected to horizontal base excitations (Frandsen, 2005; Tait et al., 2005). The publication of Ibrahim et al. (2001) offers a broad overview of sloshing dynamics, including both linear and nonlinear analysis, with emphasis on vertical cylindrical and rectangular tanks. However, containers of other geometries, such as spheres, have received little attention, despite the fact that they have several industrial applications. The effects of liquid sloshing in spherical containers were studied, for example, by McIver (1989), Evans and Linton (1993), McIver and McIver (1993) and Papaspyrou et al. (2003, 2004). Moreover, two recent studies that develop a mathematical model for calculating linear sloshing effects in the dynamical response of horizontal cylindrical and spherical liquid containers under earthquake excitation were presented by Karamanos et al. (2006) and Patkas and Karamanos (2007).

On the other hand, the amount of works concerning seismic behaviour of elevated tanks is quite limited, compared with the large number of publications on ground-supported containers (Haroun and Ellaithy, 1985; Rai, 2002). An important work that describes the dynamical behaviour of a water tower by a continuous model was attributed to Dieterman (1986). From this approach, the author derived analytically a coupled lumped-impedance model of liquid-support structure system, which was confirmed by several measurements. Similarly, Dieterman (1993) proposed a complementary model by which the effects of liquid and foundation on the structural dynamics can be evaluated integrally. By a sensitivity study, the work provides valuable evidence for the influence of liquid and soil parameters on the dynamical behaviour of the system. Recently, a paper on simplified seismic analysis procedures for elevated cylindrical tanks considering fluid–structure–soil interaction by different models was reported by Livaoğlu and Doğangün (2006). Moreover, Livaoğlu and Doğangün (2007) investigated the embedment effects on the seismic response of fluid–elevated tank–foundation–soil systems in which the fluid–structure interaction was taken into account using a Lagrangian fluid FE approximation. The seismic performance of elevated cylindrical tanks damaged during the 1999 Kocaeli earthquake in Turkey analyzed by dynamical analysis using a simplified three-mass model were reported by Sezen et al. (2008).

For the particular case of the dynamical behaviour of elevated spherical tanks under lateral excitation, only a few works have been found. Drosos et al. (2005) investigated numerically the seismic response of a typical spherical liquid storage tank equipped with a nonlinear viscous bracing system. In order to quantify the response reduction due to seismic isolation, a number of parametric nonlinear time-history analyses on a simplified Housner model of a typical sphere equipped with different types of isolation systems (lead rubber bearings, LRB, and high damping rubber bearings, HDRB) have been carried out by Bergamo et al. (2006). On a similar simplified model, a retrofit design scheme utilizing energy-dissipating braces instead of the existing ones has been proposed by Castellano et al. (2006). Karamanos et al. (2006) proposed a methodology based on a "convective-impulsive" decomposition of the liquid-vessel motion and a semi-analytical solution of sloshing in non-deformable containers by which the seismic forces can be estimated. Additionally, the effects of the support structure flexibility are also considered.

Because most of the investigations on the dynamical behaviour, taking account the sloshing of spherical liquid containers are numerical, special attention must be paid to experimental studies which are essential to validate analytical and numerical formulations. Thus, the main objective of this paper is to gain insight in the physical response of elevated spherical tanks, with emphasis in the frequencies and mode shape changes due to the stepwise increase in the liquid level. The major interest is to study the changes in frequencies and mode shapes which are principally excited by horizontal base motions, disregarding specific modes of the liquid and shell (circumferential wavy patterns). The natural frequencies of the system in the range of 1–5 Hz for different water levels were determined by free vibration tests with small vibration amplitudes. Also, by means of a linear steady-state harmonic analysis, the frequencies and mode shapes were computed with a FE model. The agreement between numerical and experimental results is excellent.

2. Dynamic tests

2.1. Case study

Sometimes, the comparison of numerical results obtained with different theories leads to a dilemma and another point of view is necessary to resolve the differences. As pointed out in Section 1, experimental evidence on the vibration of elevated spherical tanks is rather scarce in the literature. Therefore, a series of free vibration tests concerning this structural typology were carried out.

The plastic spherical shells tested have a radius $R = 81.3$ mm, wall thickness $e = 3$ mm and mass density $\rho_s = 980$ kg/m^3 (see Fig. 1). It was mentioned before that it was not necessary to include the flexibility of the shell (shell wavy modes), because this study emphasizes the behaviour of the structure as a whole. The sphere is supported by two legs with a length $L = 260$ mm, a cross-section of 3×35 mm^2, and the following material characteristics: Young's modulus $E = 2.35$ GPa, Poisson ratio $\upsilon = 0.3$ and mass density $\rho_l = 980$ kg/m^3. The contained liquid is water with a density of 1000 kg/m^3 and bulk modulus of 2.25 GPa. The legs were clamped at the base. The dimensions of the model are according to the available laboratory equipment (instrumentation, anchorage systems, etc.).

2.2. Experimental set-up and instrumentation

In order to determine the natural frequencies, free vibration tests were conducted. The structure was excited by two different means, an impact on the base of the structure and an initial displacement on the equator of the sphere. For each liquid level, four time histories of the structural response on the equator of sphere were measured by means of a PCB Piezotronics capacitive accelerometer (700 mV/g). A data acquisition board Computerboards PCM-DAS16D/16 of 16 bit of resolution and a maximum conversion time of 10 μs (100 kHz) was mounted on a notebook computer in order to record and process the signals by means of the program HP VEE (1998). The signals were recorded with a total number of points, $N = 10\,000$ and sampling rate, $n = 500$ sps. An algorithm to obtain and process the data was programmed in the environment HP VEE (1998) and the spectral density (spectrum) of the signals was estimated using the Welch's method (Ewins, 2000).

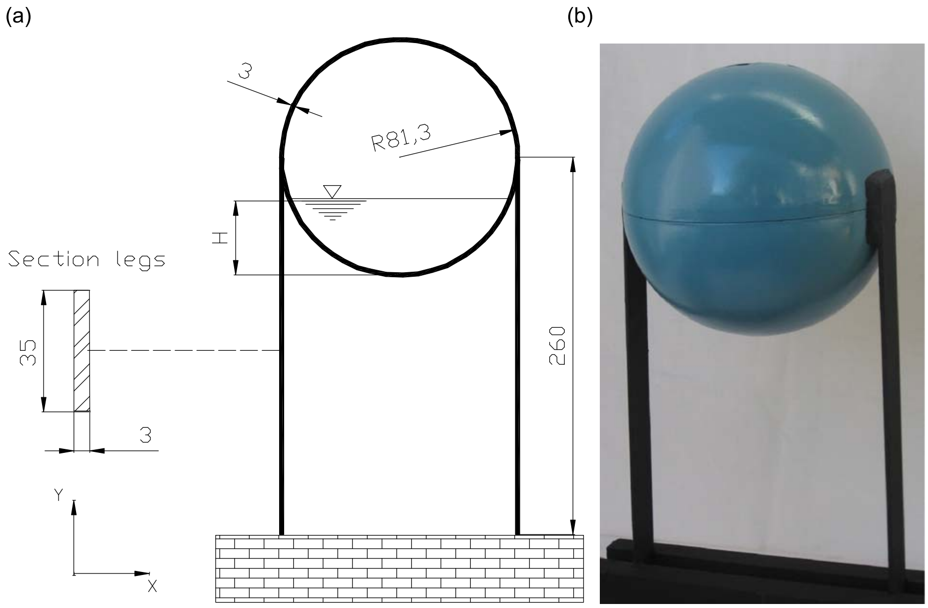

Fig. 1. Spherical container: (a) sketch; dimensions in mm and (b) photograph.

2.3. Dynamical response and experimental results

A total of 16 free vibration tests were conducted at eight gradually increasing water surface levels defined by the ratio between height of the free surface of water and radius of the container, from $H/R = 0$ (empty) to $H/R = 2$ (full) (see Table 1). A general view of the average natural frequencies measured in the range of 1–5 Hz (peaks of spectrum) for each water level is shown in Fig. 2. The dependence of resonance frequencies on the container filling is shown in Fig. 3.

In the range of 1–5 Hz, Figs. 2 and 3 provide valuable evidence on the influence of rising liquid level on the natural frequencies.

For an empty container, only one frequency of 4.8 Hz was identified corresponding to a structural mode shape, as depicted in Fig. 2. Up to liquid filling of 20%, ($H/R = 0.58$), there is a major natural frequency that dominates the dynamical behaviour and it is mostly structural. From a filling of 10%, two lower frequencies appear which became more important as the liquid level increases. The intermediate frequency seems to correspond most to a sloshing liquid mass, because it is important for around half-full tank when the free liquid surface has maximum area and it disappears for an empty and full container. Between 15% and 70% filling ($H/R = 0.49$ and 1.3, respectively) the dynamical behaviour is characterized by three resonant frequencies. This feature shows that the sloshing depends strongly on the container shape (Patkas and Karamanos, 2007) and it confirms that, for elevated spherical containers, at least three mode shapes are necessary to describe the free vibration response, unlike the two mode shapes used in cylindrical

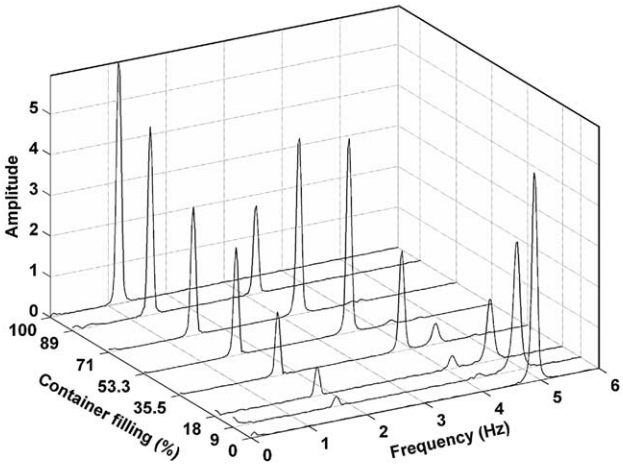

Fig. 2. Spectrum of measured free vibration response, for 8 water surface levels.

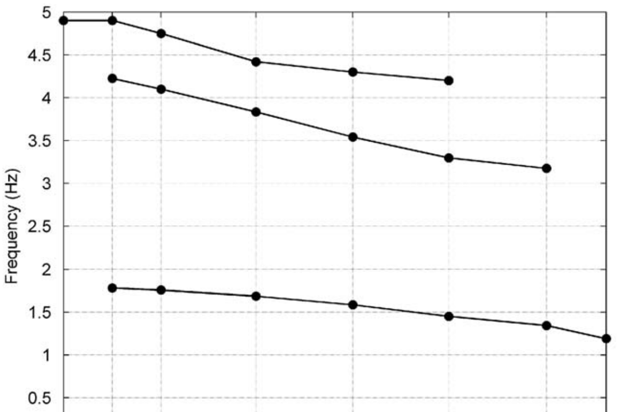

Fig. 3. Natural frequencies in the range of 1–5 Hz for 8 water surface levels.

containers (Housner, 1963; Tedesco et al., 1987; Cho et al., 2001). However, from about a water level $H/R = 0.8$ (35% filling) the highest frequency is of less importance than the others. In relatively high water surface level ($H/R = 1.3$ or 70% filling), the highest frequency disappears; finally, when the container is full, the only frequency that remains is the lowest one at 1.2 Hz, corresponding again to a structural mode shape similar to a solid body in which the sphere contains the total coupled mass of liquid with no sloshing.

It is important to note that, due to the interaction between fluid and structure, the resonant frequency dependency on the fluid level has a particular characteristic as shown in Fig. 3. In the range of interest, the lower frequency decreases in a uniform way as the fluid level increases, whereas the highest frequency (''structural'' frequency) displays a stepwise profile with an important drop between levels of 10% and 35% filling due to a strong coupling of liquid and structure.

3. Numerical study

3.1. FE model

Fluid–structure interaction can be considered using different approaches such as simplified methods with Housner's two-mass representation (Housner, 1963), multi-mass representation (Bauer, 1964), added mass in a ''solid'' Finite Element Model (FEM), and provisions of the design codes (Eurocode-8, 2003, API 650, 1998 or ACI 350.3, 2001). A comparison and evaluation of these methods are presented by Livaoğlu and Doğangün (2006). An important study on mechanical systems modelling for fluid–structure interaction was published by Axisa and Antunes (2007). For an exhaustive analysis and more complex models incorporating Lagrangian (Bennet, 2006), Eulerian (Angrand, 1985), and Lagrangian–Eulerian approaches in Finite Element Method formulations should be used (Zienkiewicz and Bettes, 1978; Wilson and Khalvati, 1983; Wang, 2008; Donea and Huerta, 2002). In this study, a linear harmonic response analysis, performed in the ANSYS (1992) Finite Element program, from a detailed model (Fig. 4) including the effect of liquid–structure interaction based on a Lagrangian approach is adopted. The spherical shell is modelled by four-node shell elements with six degrees of freedom per node and the supporting columns by two-node frame elements with six degrees of freedom per node. The eight-node solid–fluid element with three degrees of freedom per node has been chosen to model the inviscid liquid contained in the sphere without net flow rate. The finite element formulation allows acceleration effects such as sloshing. In order to satisfy the continuity conditions between the fluid and solid shell at the spherical boundary, the ''coincident'' nodes of the fluid and shell elements are constrained to be coupled in the direction normal to the interface, while relative motions are allowed to occur in the tangential directions.

3.2. Steady-state harmonic response analysis

In order to obtain the modal shapes and eigenfrequencies, two alternatives are normally used, modal (eigenvalue) analysis and steady-state harmonic response analysis. In this case, the first procedure results in time-consuming and computationally expensive analysis due to a lot of strictly liquid modes, which make difficult to search the modes where the fluid is strongly coupled with the structure; therefore, the latter analysis was adopted. The fluid–structure system analysis, was based on the evaluation of the Accelerance Function (i.e., the magnitude of Frequency Response Function—FRF) defined as the ratio between the acceleration at a node located at the equator of the sphere and the force at the base, both parameters computed along the X-axis (see Fig. 1). In this way, only the concerned frequencies and modal shapes in the range of 1–5 Hz were determined. Fig. 5 shows the numerical FRF, where the peaks correspond to the resonance eigenfrequencies of the system for the same water surface levels considered in Section 2.3. The agreement achieved between numerical (Fig. 5) and experimental (Fig. 2) spectra is very good.

For an elevated spherical tank with 35.6% and 71.1% liquid filling, Figs. 6 and 7 show the three modal shapes. It is important to note that the corresponding modes for different liquid levels from $H/R = 0.55$ (18% filling) to $H/R = 1.6$ (90% filling) preserve similar profiles.

An analysis of the mode shapes of the dynamical system in the range of 1–5 Hz (Figs. 6 and 7) shows that: the first vibration mode shape corresponding to the lowest frequency displays a strong coupling between the structure and most of the liquid mass due to the oscillating ''pendular'' motion in-phase with the structure, in which the free surface remains plane. This mode acquires importance in the dynamical behaviour of the system as the water levels increases from about 15% filling, being most important when the container is nearly full. The second mode shape (intermediate frequency) shows a marked antisymmetric slosh wave, which has a positive peak at one side and a negative peak at the other. The slosh wave and structure move out-of-phase, i.e., if the positive peak grows up to the left, the structure moves to the right and viceversa. In nearly half-full spherical containers, when the free liquid surface has maximum area, this mode has considerable participation on the free vibration structural response, and it is

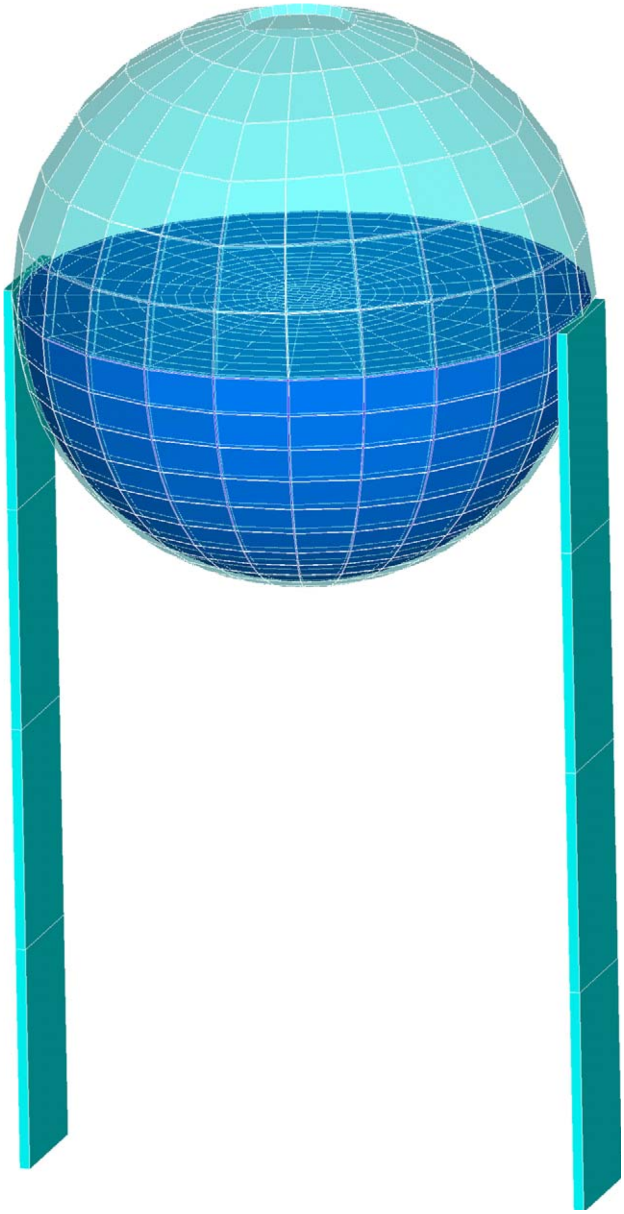

Fig. 4. Finite Element model.

insignificant for extreme conditions (nearly full and empty container). The third mode shape, mostly structural (highest frequency) makes an important contribution to the free vibration response up to liquid levels of around a filling of 35%.

Summarizing, for a container empty and up to liquid filling of 20% the third vibration mode (highest frequency) is mostly structural and it dominates the dynamical behaviour. The small liquid mass moves as a pendulum out-of-phase with the structure. When the container is around half-full, the second vibration mode (intermediate frequency) is the most important. The lower liquid mass translates jointly to the structure with little pendular motion, and the upper liquid mass presents a marked antisymmetric slosh wave. This mode shape is distinguished predominantly by a sloshing liquid mass. An oscillating pendular motion of most of the liquid mass in-phase with the translation of the structure characterizes the first mode shape (lowest frequency). This vibration mode controls the free response from a filling larger than 80%.

Note that, unlike the impulsive (bulging) mass concept for cylindrical containers (Housner, 1963; Tedesco et al., 1987; Cho et al., 2001), in the case of spherical containers most of the lower liquid mass does not move rigidly attached to the

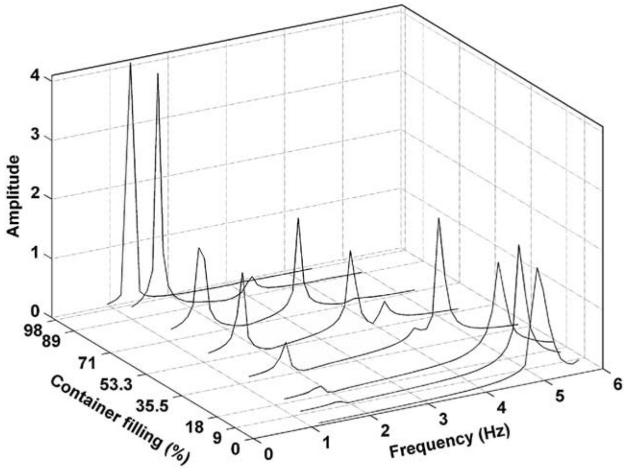

Fig. 5. FRF from FE model, for 8 water surface levels of spherical container.

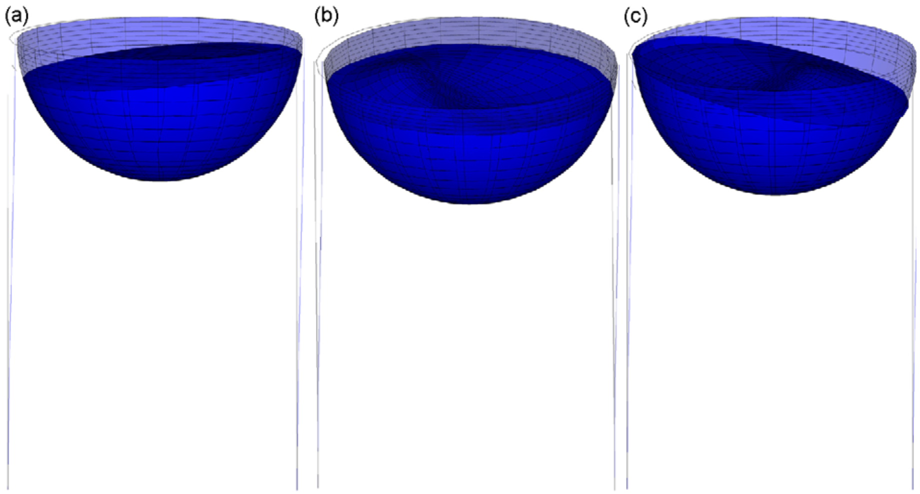

Fig. 6. Mode shapes for water surface level of $H/R = 0.81$ (35.6% filling): (a) 1.7 Hz; (b) 3.8 Hz; (c) 4.2 Hz.

container walls; but it oscillates as a pendulum in-phase or out-of-phase with the translation motion of the structure. In this context, it would be incorrect to name this mass as impulsive because it involves both translational and pendular motions. Then, in connection with vibration of the spherical container system, in this paper the following names are proposed for the three characteristic masses and associated frequencies: *structural* (translational), *sloshing* (convecting) and *pendular*.

From the above discussion it is clear that to describe the dynamical behaviour of the system, by a simplified mechanical model, three essentially independent mass-motions are necessary: *translation*, *sloshing* and *pendular motions*, and therefore a minimum of three degrees of freedom should be considered.

4. Sloshing effects

In order to investigate the effect of sloshing on the structural response of this type of containers, two case studies were analyzed: (a) the contained liquid, in this case water, was considered with its true properties (bulk modulus, density and boundary conditions as indicated above) and (b) the mass of the liquid was assumed as a rigid solid block (model without sloshing, but the same boundary conditions). The FRF of both cases, with a container 53% full (liquid level $H/R = 1.044$) are shown in Fig. 8. It was found that, if the sloshing effect is ignored (case (b)), the two highest frequencies do not appear and the lower one suffers a significant shift (from 1.65 to 2.1 Hz).

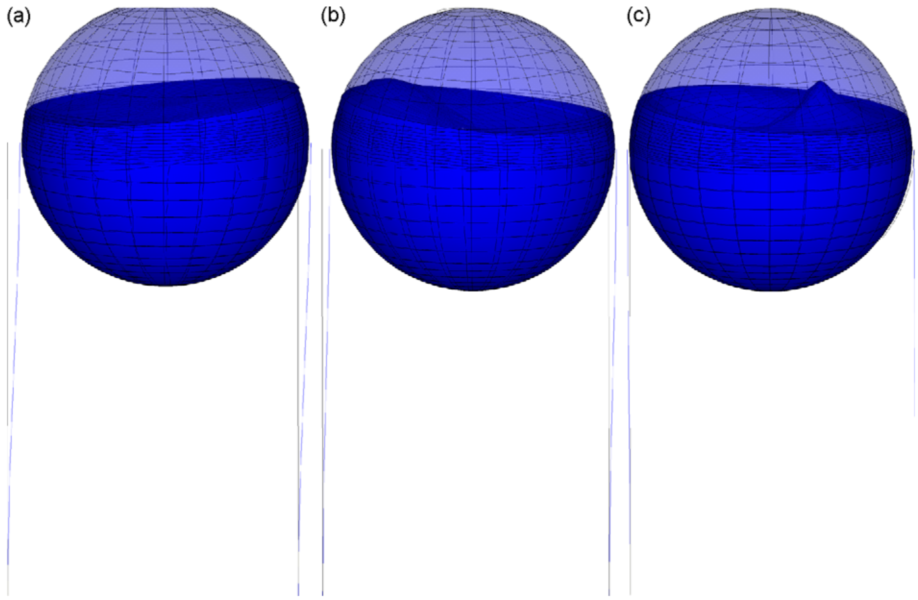

Fig. 7. Mode shapes for water surface level of $H/R = 1.29$ (71.1% filling): (a) 1.6 Hz; (b) 3.3 Hz; (c) 4.15 Hz.

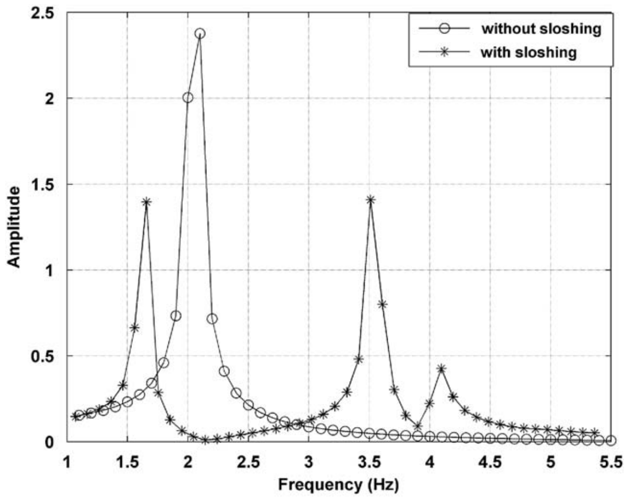

Fig. 8. Sloshing effects. Container filling of 53%.

In view of the explanations given in Sections 2.3, 3.2 and 4, it seems clear to infer that, although for design purposes of cylindrical containers, in which the sloshing is ignored or partially represented, a simplified single-mass or a two-lumped-mass model is satisfactory (Sezen et al., 2008), for thorough dynamical analysis or for damage assessment of elevated spherical container this hypothesis is not valid and more elaborate mechanical models that depict the overall dynamical behaviour of the system in a wide range of liquid surface levels are necessary. A similar statement was previously suggested by Livaoğlu and Doğangün (2006).

5. Comparison of experimental and numerical results

The natural frequencies evaluated from the measured free vibration response and the corresponding ones computed from FE model, for increasing water level H/R, are shown in Fig. 9.

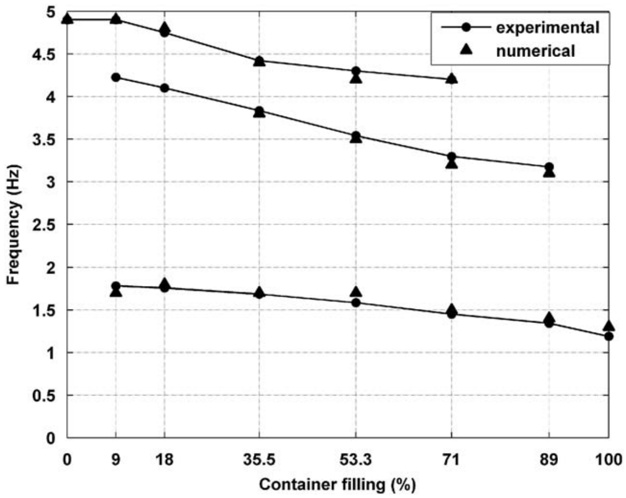

Fig. 9. The experimental (-●-) and numerical (-▲-) natural frequencies in the range of 1–5 Hz for 8 water surface levels.

Table 1
Comparison between experimental and numerical eigenfrequencies.

Volume of sphere = 2250 cm^3; radius = 81.3 mm					
H/R	Filling (%)	Mode	Frequency (Hz)		
			Experimental	Model	Diff. (%)
0.0	0.0 (empty)	1	–	–	–
		2	–	–	–
		3	4.9	4.9(4.89)[a]	0.00(0.20)
0.37	8.9	1	1.782	1.7	−4.60
		2	4.225	–	–
		3	4.9	4.9	0.00
0.54	17.8	1	1.758	1.8	2.40
		2	4.10	–	–
		3	4.75	4.8	1.05
0.81	35.6	1	1.685	1.7	0.90
		2	3.833	3.8	−0.86
		3	4.42	4.4	−0.45
1.044	53.3	1	1.585	1.7	7.25
		2	3.54	3.5	−1.13
		3	4.3	4.2	−2.30
1.29	71.1	1	1.45	1.5	3.45
		2	3.296	3.2	−2.90
		3	4.2	4.2	0.00
1.59	88.9	1	1.343	1.4	4.24
		2	3.17	3.1	−2.33
		3	–	–	–
2.0	100 (full)	1	1.19	1.3 (1.16)[a]	9.24 (2.50)
		2	–	–	–
		3	–	–	–

[a]Frequencies calculated by Eq. (1).

Fig. 9 shows a slight difference between experimental and numerical values of the natural frequencies for each liquid level in the range of 1–5 Hz. A more precise comparison can be made by the local maximum values (peaks) of spectra as shown in Table 1. Insignificant values (very low peaks) are not indicated. The relative differences included in Table 1 are defined by $d_{ij} = 100(f_i - f_j)/f_i$, where i corresponds to the measured frequencies and j to the frequencies determined by the numerical model. The greatest differences are within 10% and the average difference is 2.5%. The agreement between the experimental and numerical results is very good.

Because the fundamental mode shapes of the empty and full container cases are of the "shear type", the natural frequencies could be easily approximated using the following expression (Den Hartog, 1956):

$$f_e = \frac{1}{2\pi}\sqrt{\frac{2k_e}{m_e + 0.23m_l}}\sqrt{\left(1 - \frac{P_o}{P_{cr}}\right)}, \tag{1}$$

with

$$k_e = 12\frac{EI}{L^3}. \tag{2}$$

In the equation above, f_e is the fundamental frequency of empty container, k_e the "shear" stiffness of one leg, m_e the empty spherical container mass, m_l the mass of the legs, EI the flexural stiffness, L the length of the legs, P_o the leg axial load, and P_{cr} the first elastic critical load.

The agreement between analytical and measured fundamental eigenfrequencies for empty and full container cases is very good as showed in Table 1.

6. Conclusions

The present paper investigates the overall dynamical response of coupled fluid–structure systems of elevated spherical tanks subjected to horizontal base motion. First, impact tests (with small vibration amplitudes) on an experimental model were performed, identifying the natural frequencies of the vibration modes that contribute to the dynamical response in the range of 1–5 Hz, for different liquid levels. Next, a numerical model that takes into account the coupling between fluid and structure was developed and validated against the experimental results. A very good agreement between experimental and numerical results was obtained.

As expected, the frequencies of the partially fluid-filled spherical container decrease with increasing fluid level. Due to the interaction between fluid and structure, the dependency of the natural frequencies on the fluid level has a particular characteristic. The lower measured frequencies, in the range of interest, decrease in a uniform way as the fluid level increases, whereas the highest frequency (structural frequency) displays a stepwise profile with an important drop between the levels of 10% and 35% filling.

The results indicate that sloshing has a significant effect on the dynamical characteristics of the system; thus, in cases where an accurate dynamical analysis is required, the sloshing should be considered. Moreover, it is clear that, systems such as elevated spherical tanks display three essentially independent mass-motions due to the impulsive mass is not rigidly attached to container walls as it is assumed in cylindrical containers. Therefore, a simplified model of two lumped-masses is insufficient to describe the overall dynamical behaviour of the system for a wide range of liquid levels, and a minimum of three masses (three degrees of freedom) should be considered. Names for these three mass have been proposed in this paper.

These conclusions help to understand the changes in the dynamical characteristics caused by container shape and fluid level and give rise to the development of simplified mechanical lumped-mass models and damage structural assessment methods based on frequency changes.

Acknowledgments

The financial support of CONICET and Universidad Nacional de Cuyo is gratefully acknowledged. Special thanks are extended to the reviewers of the first version of the paper; their useful suggestions improved the work.

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